Exempel

Lös systemet av partiella differentialekvationer i $\mathbb{R}^3$:

$$\begin{cases} w'_x = 2xz + y, \\ w'_y = x + 3y^2 z, \\ w'_z = x^2 + y^3 + 1. \end{cases}$$

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SVAR: $w = x^2z + xy + y^3z + z + c.$