Exempel

Låt

$$\begin{cases} u = x^2 + y \\ v = \sin z \\ w = xyz. \end{cases}$$

Bestäm funktionalmatrisen $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$ och funktionaldeterminanten $\dfrac{d(u,v,w)}{d(x,y,z)}$.
Visa även att $(x,y,z) \mapsto (u,v,w)$ har en $\mathcal{C}^1$ invers i någon omgivning till $(x,y,z) = (1,1,\pi)$.

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Eftersom funktionerna är av klass $\mathcal{C}^1$ och

$$\frac{d(u,v,w)}{d(x,y,z)}(1,1,\pi) = \pi \neq 0$$

ger inversa funktionssatsen att avbildningen är lokalt inverterbar med $\mathcal{C}^1$ invers.