Exempel

Visa att ekvationen

$$xz - x^2y = \mathrm{e}^z - 1$$

lokalt i någon omgivning till $(x, y, z) = (0, 0, 0)$ bestämmer z unikt som funktion av (x, y) (d.v.s. lokalt är ytan en graf $z = z(x, y)$). Beräkna även z'_x och z'_y för denna funktion.

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