Exempel

Beräkna

$$\iint_D |xy|\, dxdy$$

där $D = \{(x,y) \in \mathbb{R}^2 : -1 \leq x \leq 1, 0 \leq y \leq 2\}.$

Lösning

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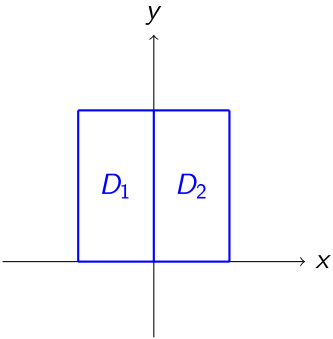

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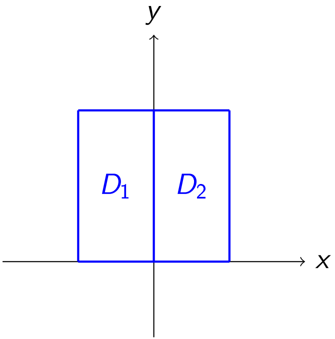

$$D = D_1 \cup D_2 =$$

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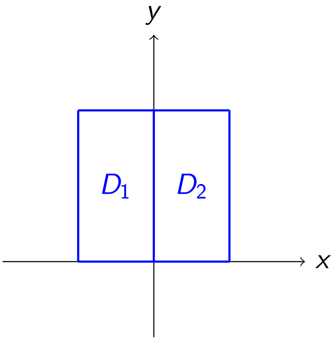

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På D_1 gäller $|xy| = -xy$ och på D_2 gäller $|xy| = xy$.

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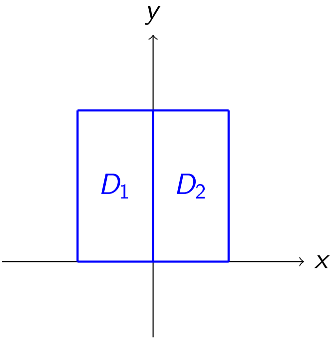

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$$\iint_{D_1} (-xy)dxdy + \iint_{D_2} xydxdy =$$
$$\int_0^2 \left(\int_{-1}^0 (-xy)dx \right) dy + \int_0^2 \left(\int_0^1 xydx \right) dy =$$
$$= -\int_0^2 \left[\frac{x^2 y}{2} \right]_{x=-1}^0 dy + \int_0^2 \left[\frac{x^2 y}{2} \right]_{x=0}^1 dy =$$
$$-\int_0^2 -\frac{y}{2} dy + \int_0^2 \frac{y}{2} dy = \ldots = 2.$$