Exempel

Beräkna

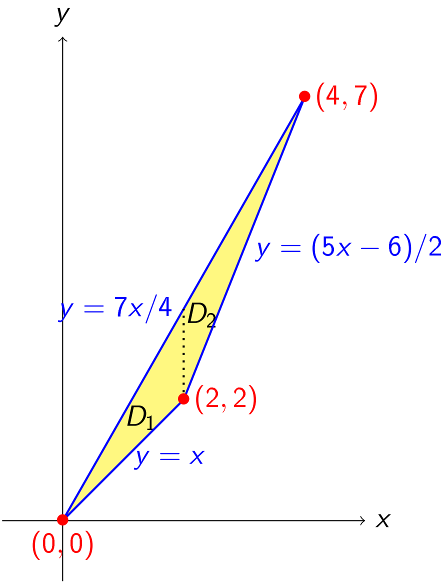
$$\iint_D y dx dy$$

där

(a) D är triangeln med hörn i $(0,0), (2,2)$ och $(4,7)$.

(b) $D = \{(x,y) : x^2 + y^2 \leq 1, x^2 + 4y^2 \geq 1\}$.

Lösning (a)



Lösning (a)

$$D = D_1 \cup D_2$$

där

$$\begin{aligned} D_1 &= \{(x,y) : 0 \le x \le 2, x \le y \le 7x/4\}, \\ D_2 &= \{(x,y) : 2 \le x \le 4, (5x-6)/2 \le y \le 7x/4\}. \end{aligned}$$

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$$\iint_D y dx dy = \iint_{D_1} y dx dy + \iint_{D_2} y dx dy$$

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$$\int_0^2 \left(\int_x^{7x/4} ydy \right) dx + \int_2^4 \left(\int_{(5x-6)/2}^{7x/4} ydy \right) dx$$

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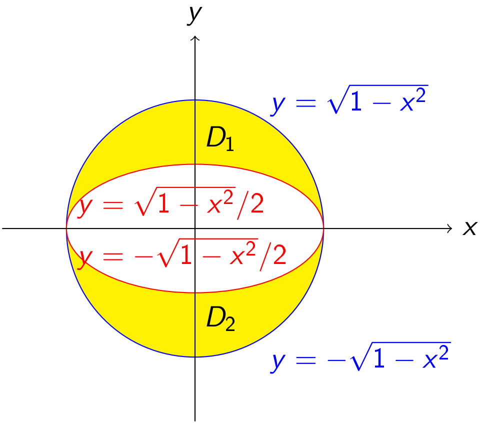
Lösning (b)

$$D = \{(x,y) : x^2 + y^2 \leq 1, x^2 + 4y^2 \geq 1\}$$



Lösning (b)

$$D = D_1 \cup D_2$$

där

$$\begin{aligned} D_1 &= \left\{(x,y) : -1 \le x \le 1, \frac{1}{2}\sqrt{1-x^2} \le y \le \sqrt{1-x^2}\right\}, \\ D_2 &= \left\{(x,y) : -1 \le x \le 1, -\sqrt{1-x^2} \le y \le -\frac{1}{2}\sqrt{1-x^2}\right\}. \end{aligned}$$

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$$\iint_D y\,dx\,dy = \iint_{D_1} y\,dx\,dy + \iint_{D_2} y\,dx\,dy =$$
$$\int_{-1}^{1}\left(\int_{\sqrt{1-x^2}/2}^{\sqrt{1-x^2}} y\,dy\right)dx + \int_{-1}^{1}\left(\int_{-\sqrt{1-x^2}}^{-\sqrt{1-x^2}/2} y\,dy\right)dx$$

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$$\iint_D ydxdy = \iint_{D_1} ydxdy + \iint_{D_2} ydxdy =$$
$$\int_{-1}^{1}\left(\int_{\sqrt{1-x^2}/2}^{\sqrt{1-x^2}} ydy\right)dx + \int_{-1}^{1}\left(\int_{-\sqrt{1-x^2}}^{-\sqrt{1-x^2}/2} ydy\right)dx = \ldots = 0.$$