Matlab code

Logistic Lotka-Volterra

```
function dZdt = logLV(~, Z, a, b, c, d, K)
    x = Z(1);
    y = Z(2);

    dxdt = a*x*(1-x/K)-b*x*y;
    dydt = -c*y+d*x*y;

    dZdt = [dxdt; dydt];
end
```

Saved as logLV.m

```
% Logistic Lotka-Volterra Phase Plane Plot with Vector Field,
Trajectories,  Equilibria, and x and y plotted as functions of time
%
--------------------------------------------------------------------
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% Parameters
a=2;
b=4;
c=1;
d=1;
K=1.5;

% Phase plane grid for vector field
[Sg, Ig] = meshgrid(linspace(0,2,20), linspace(0,1,20));
Ug = zeros(size(Sg));
Vg = zeros(size(Ig));

for i = 1:numel(Sg)
    x = Sg(i);
    y = Ig(i);
    dZ = logLV(0,[x;y],a,b,c,d,K);
    Ug(i) = dZ(1);
    Vg(i) = dZ(2);
end

% Normalize vectors for clarity
L = sqrt(Ug.^2 + Vg.^2);
Ug = Ug./L; Vg = Vg./L;

figure; subplot(2,1,1); hold on;

% Plot vector field
quiver(Sg, Ig, Ug, Vg, 0.5, 'k','LineWidth',1,'MaxHeadSize',2);

% Initial conditions for three trajectories
ICs = [0.2 0.5;
       1.8 0.25;
       1.0 0.8];

colors = lines(size(ICs,1));

% Plot trajectories
for k = 1:size(ICs,1)
    [t, Z] = ode45(@(t,Z) logLV(t,Z,a,b,c,d,K), [0 100], ICs(k,:));

    plot(Z(:,1), Z(:,2), 'Color', colors(k,:), 'LineWidth', 2);
    plot(Z(1,1), Z(1,2), 'o', 'Color', colors(k,:), ...
        'MarkerFaceColor', colors(k,:), 'MarkerSize',6);
end

% Equilibrium
xx = c/d;
yy = a/b-a*c/(b*d*K)
plot(xx, yy, 'ro', 'MarkerFaceColor','green','MarkerSize',8);

xlabel('x');
ylabel('y');
title('Phase Plane of logistic LV Model with Vector Field and
Equilibria');
grid on; box on;

% Time Series of x(t) and y(t)

subplot(2,1,2); hold on;

for k = 1:size(ICs,1)
    [t, Z] = ode45(@(t,Z) logLV(t,Z,a,b,c,d,K), [0 20], ICs(k,:));
    plot(t, Z(:,1), '--', 'Color', colors(k,:), 'LineWidth', 2); %
S(t)
    plot(t, Z(:,2), '-',  'Color', colors(k,:), 'LineWidth', 2); %
I(t)
end

xlabel('Time');
ylabel('Populations');
title('Time Series: x(t) and y(t)');
legend({'x(t) IC1','y(t) IC1','x(t) IC2','y(t) IC2','x(t) IC3','y(t)
IC3'}, ...
        'Location','best');
grid on; box on;
```
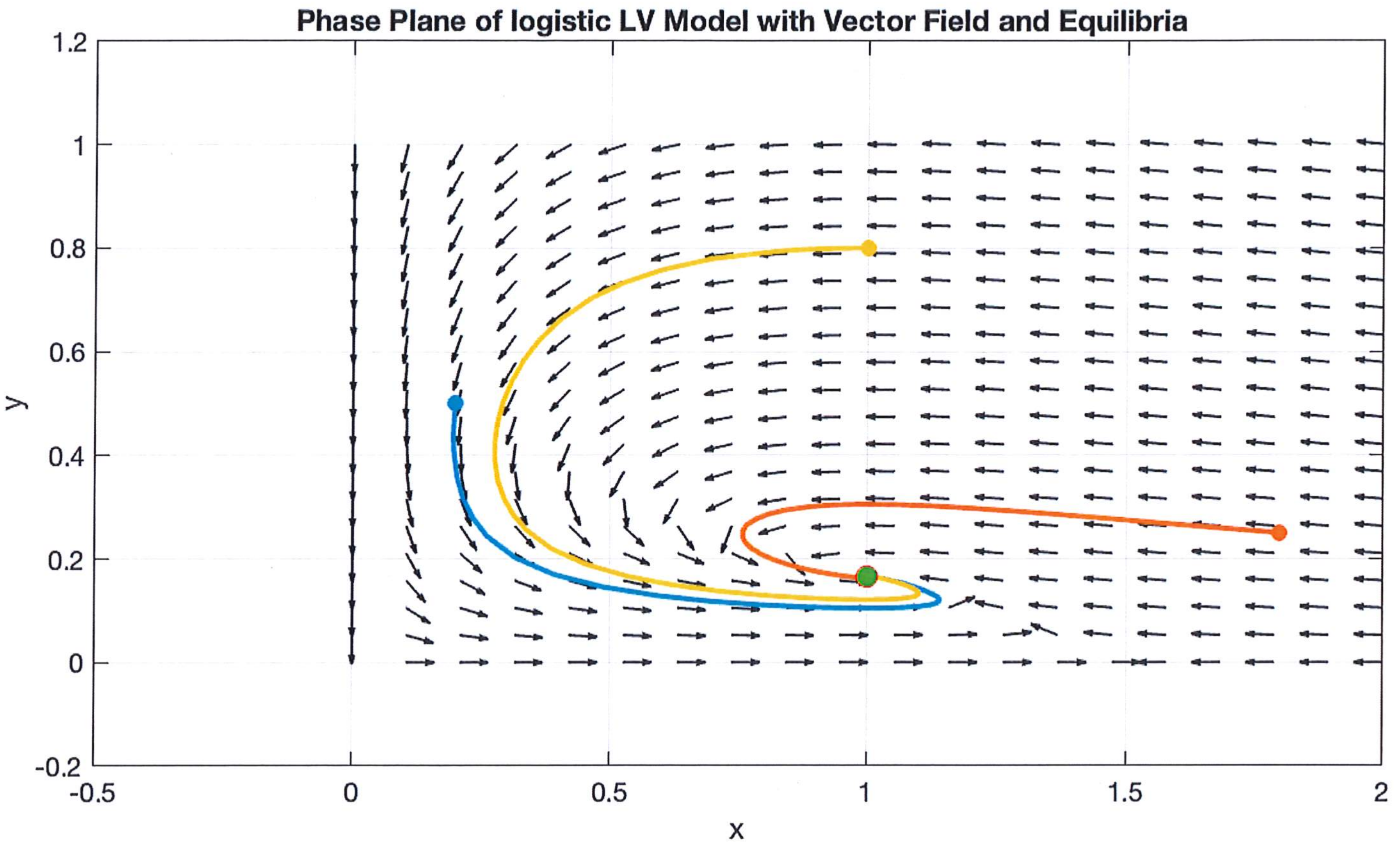
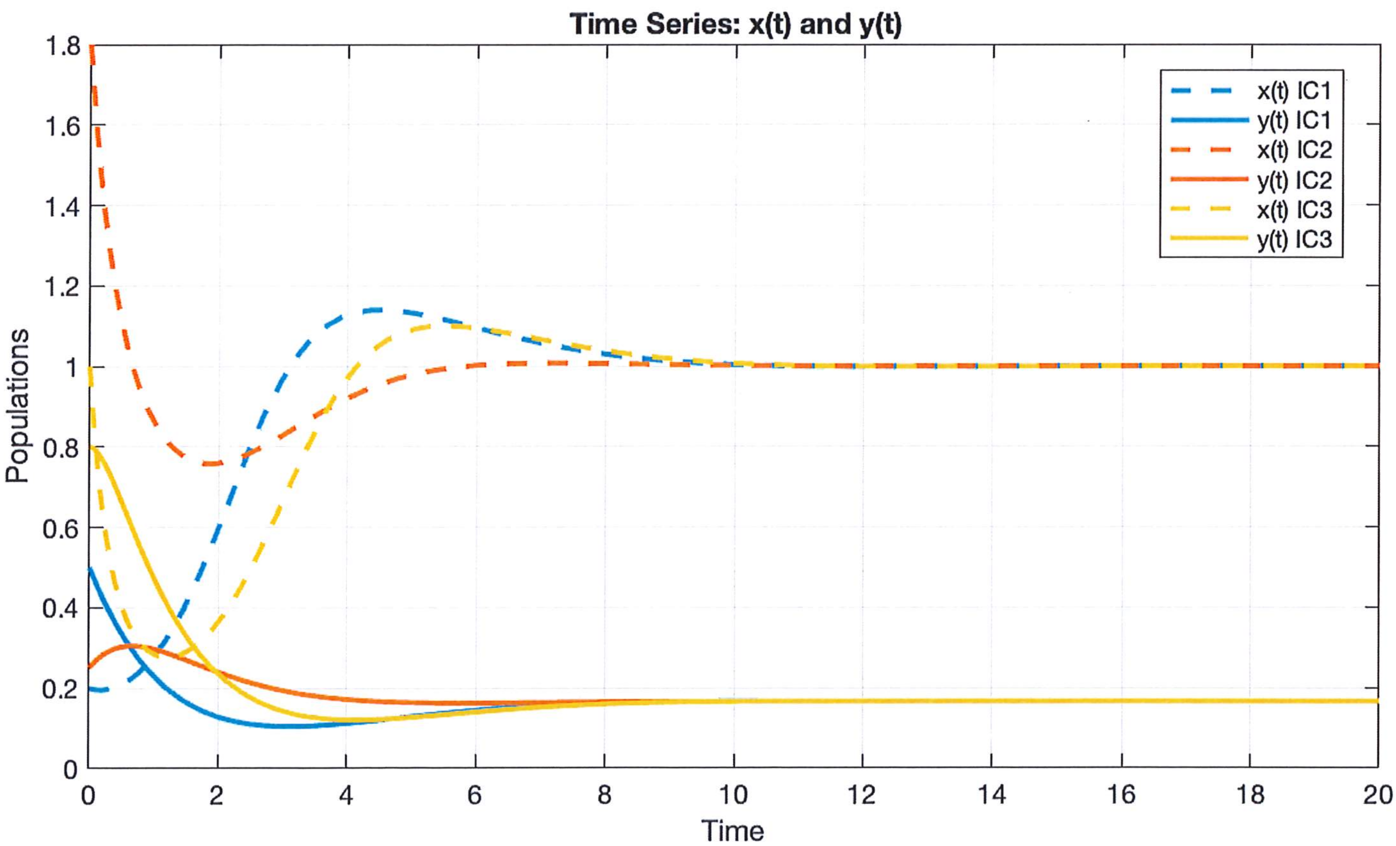



$a=2$, $b=4$, $c=d=1$, $K=1.5$, $(\bar{x},\bar{y})=(1,\frac{1}{6})$



Matlab code
SIRS

```
function dYdt = sirs(~, Y, beta, gamma, nu)
    S = Y(1);
    I = Y(2);
    R = Y(3);
    N = S + I + R;  % constant

    dSdt = -beta*S*I + gamma*R;
    dIdt = beta*S*I - nu*I;
    dRdt = nu*I - gamma*R;

    dYdt = [dSdt; dIdt; dRdt];
end
```

Saved as sirs.m

```
% SIRS Phase Plane Plot with Vector Field, Trajectories, Equilibria,
% and S and I plotted as functions of time
%
----------------------------------------------------------------------
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% Parameters
beta = 1;
gamma = 4;
nu = 4;
N = 10;

% Phase plane grid for vector field
[Sg, Ig] = meshgrid(linspace(0,10,20), linspace(0,10,20));
Ug = zeros(size(Sg));
Vg = zeros(size(Ig));

for i = 1:numel(Sg)
    S = Sg(i);
    I = Ig(i);
    R = N - S - I;
    if R >= 0
        dY = sirs(0,[S;I;R],beta,gamma,nu);
        Ug(i) = dY(1);
        Vg(i) = dY(2);
    else
        Ug(i) = NaN; Vg(i) = NaN; % outside feasible region
    end
end

% Normalize vectors for clarity
L = sqrt(Ug.^2 + Vg.^2);
Ug = Ug./L; Vg = Vg./L;

figure; subplot(2,1,1); hold on;

% Plot vector field
quiver(Sg, Ig, Ug, Vg, 0.5, 'k','LineWidth',1,'MaxHeadSize',2);

% Initial conditions for three trajectories
ICs = [9.9 0.1 0;
       2.0 0.5 7.5;
       3.0 7.0 0];

colors = lines(size(ICs,1));

% Plot trajectories
for k = 1:size(ICs,1)
    [t, Y] = ode45(@(t,Y) sirs(t,Y,beta,gamma,nu), [0 5], ICs(k,:));

    plot(Y(:,1), Y(:,2), 'Color', colors(k,:), 'LineWidth', 2);
    plot(Y(1,1), Y(1,2), 'o', 'Color', colors(k,:), ...
        'MarkerFaceColor', colors(k,:), 'MarkerSize',6);
end

% Endemic equilibrium (if R0>1)
R0 = beta*N/nu;
if R0 > 1
    II = (gamma*(N*beta - nu)) / (beta*(gamma+nu));
    SS = nu/beta;
    RR = N - SS - II;
    if II > 0 && RR > 0
        plot(SS, II, 'ro',
'MarkerFaceColor','green','MarkerSize',8);
    end
end


xlabel('S (Susceptible)');
ylabel('I (Infected)');
title('Phase Plane of SIRS Model with Vector Field and Equilibria');
grid on; box on;

%% --- Time Series of S(t) and I(t) ---

subplot(2,1,2); hold on;

for k = 1:size(ICs,1)
    [t, Y] = ode45(@(t,Y) sirs(t,Y,beta,gamma,nu), [0 5], ICs(k,:));
    plot(t, Y(:,1), '--', 'Color', colors(k,:), 'LineWidth', 2); %
S(t)
    plot(t, Y(:,2), '-',  'Color', colors(k,:), 'LineWidth', 2); %
I(t)
end

xlabel('Time');
ylabel('Population Fractions');
title('Time Series: S(t) and I(t)');
legend({'S(t) IC1','I(t) IC1','S(t) IC2','I(t) IC2','S(t) IC3','I(t)
IC3'}, ...
        'Location','best');
grid on; box on;
```

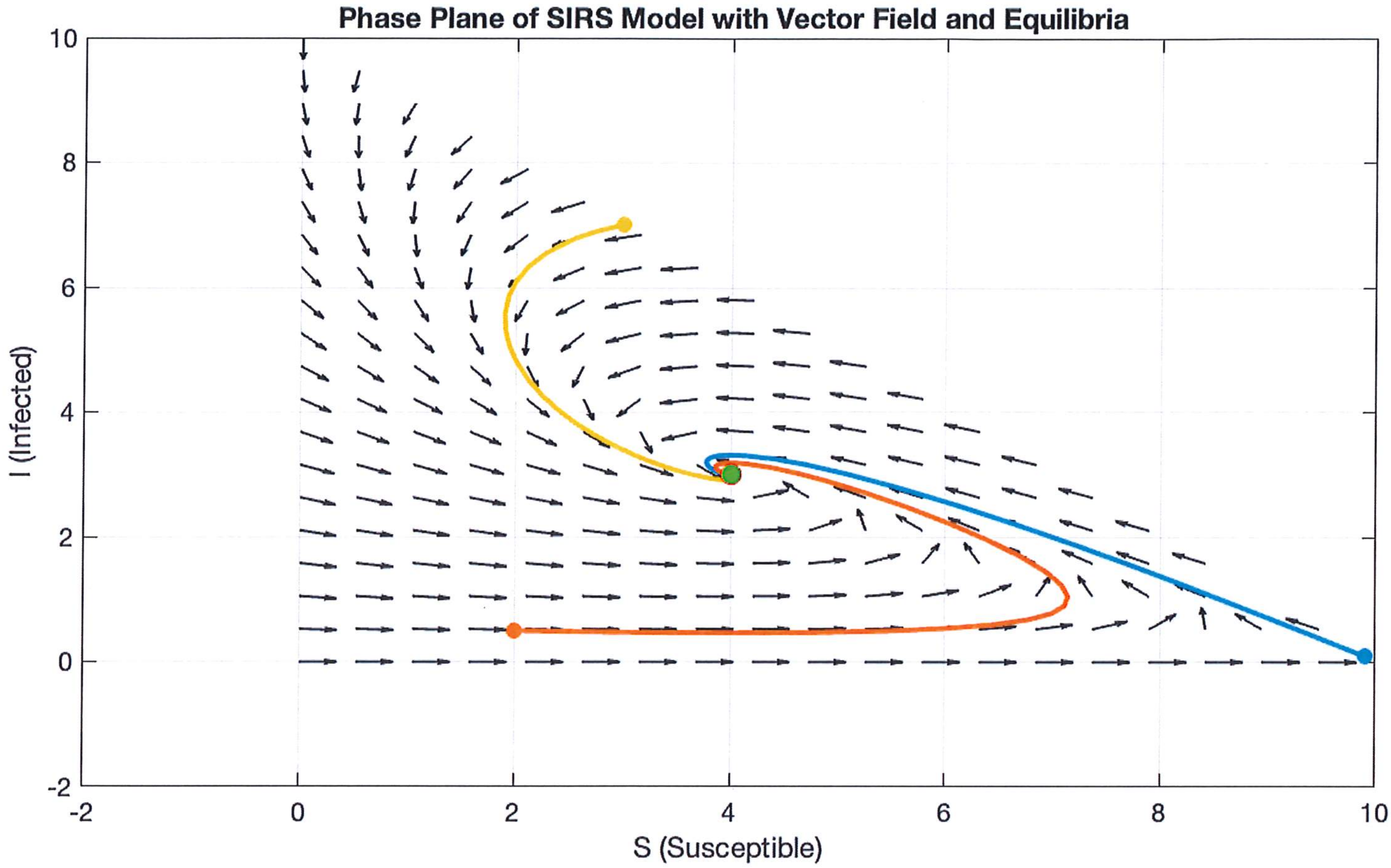

$N=10,\ \beta=1,\ \nu=\gamma=4 \quad \Rightarrow R_0=2.5,\ (\bar{S},\bar{I})=(4,3)$